Synchronization-Aware and Algorithmically-Efficient Chance Constrained Optimal Power Flow

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Review of past work: chance-constrained DC OPF

- CIGRE '09: large unexpected fluctuations in wind power can cause additional flows through the transmission system (grid)
- Large power deviations in renewables must be balanced by other sources, which may be far away
- Flow reversals may be observed – control difficult
- A solution – expand transmission capacity! Difficult (expensive), takes a long time
- Problems **already observed** when renewable penetration high

CIGRE -International Conference on Large High Voltage Electric Systems '09

- "Fluctuations" – 15-minute timespan
- Due to turbulence ("storm cut-off")
- Variation of the same order of magnitude as mean
- Most problematic when renewable penetration starts to exceed $20-30\%$
- Many countries are getting into this regime

DC-OPF:

$$\min \ c(p) \quad \text{(a quadratic)}$$

s.t.

$$B\theta = p - d \tag{1}$$

$$|\beta_{ij}(\theta_i - \theta_j)| \ \leq \ u_{ij} \quad \text{for each line } ij \tag{2}$$

$$P_g^{min} \ \leq \ p_g \ \leq \ P_g^{max} \quad \text{for each generator } g \tag{3}$$

Notation:

$p \ = \$ vector of generations $\ \in \mathcal{R}^n, \quad d \ = \$ vector of loads $\ \in \mathcal{R}^n$

$B \ \in \mathcal{R}^{n \times n}, \quad$ (bus susceptance matrix)

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How does OPF handle short-term fluctuations in **demand** (d)?
Frequency control:

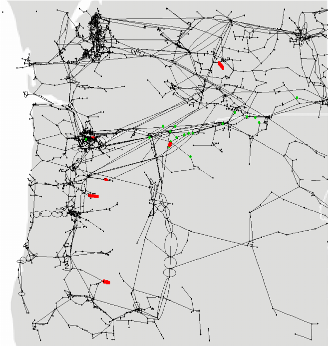
- Automatic control: primary, secondary
- Generator output varies up or down **proportionally** to **aggregate** change

How does OPF handle short-term fluctuations in renewable output?
Answer: Same mechanism, now used to handle aggregate wind power change

Experiment

Bonneville Power Administration data, Northwest US

- data on wind fluctuations at planned farms
- with standard OPF, 7 lines exceed limit $\geq 8\%$ of the time



Line trip model

summary: exceeding limit for too long is bad, but complicated

want: "fraction time a line exceeds its limit is small"

proxy: prob(violation on line i) $< \epsilon$ for each line i

Goals

- simple control
- aware of limits
- not too conservative
- computationally practicable

Control

For each generator i, two parameters:

- $\overline{p_i}$ = mean output
- α_i = response parameter

Real-time output of generator i:

$$p_i = \overline{p}_i - \alpha_i \sum_j \Delta\omega_j$$

where $\Delta\omega_j$ = change in output of renewable j (from mean).

$$\sum_i \alpha_i = 1$$

$\sim$ primary + secondary control

Computing line flows

wind power at bus i: $\mu_i + \mathbf{w}_i$

DC approximation

- $B\boldsymbol{\theta} = \overline{p} - d$
  $+(\mu + \mathbf{w} - \alpha \sum_{i \in G} \mathbf{w}_i)$
- $\boldsymbol{\theta} = B^{+}(\bar{p} - d + \mu) + B^{+}(I - \alpha e^{T})\mathbf{w}$
- flow is a linear combination of bus power injections:

$$\mathbf{f_{ij}} = \beta_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)$$

Computing line flows

$$\mathbf{f_{ij}} = \beta_{ij}\left((B_i^+ - B_j^+)^T(\bar{p} - d + \mu) + (A_i - A_j)^T \mathbf{w}\right),$$

$$A = B^+(I - \alpha e^T)$$

Given distribution of wind can calculate moments of line flows:

- $E\mathbf{f_{ij}} = \beta_{ij}(B_i^+ - B_j^+)^T(\bar{p} - d + \mu)$
- $var(\mathbf{f_{ij}}) := s_{ij}^2 \geq \beta_{ij}^2 \sum_k (A_{ik} - A_{jk})^2 \sigma_k^2$ (assuming independence)
- and higher moments if necessary

Chance constraints to deterministic constraints

- chance constraint: $P(\mathbf{f_{ij}} > f_{ij}^{max}) < \epsilon_{ij}$ **and** $P(\mathbf{f_{ij}} < -f_{ij}^{max}) < \epsilon_{ij}$
- from moments of $\mathbf{f_{ij}}$, can get conservative approximations using e.g. Chebyshev's inequality

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- from moments of $\mathbf{f_{ij}}$, can get conservative approximations using e.g. Chebyshev's inequality
- for Gaussian wind, can do better, since $\mathbf{f_{ij}}$ is Gaussian :

$$|E\mathbf{f_{ij}}| + var(\mathbf{f_{ij}})\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$$

Formulation:

Choose mean generator outputs and control to minimize expected cost, with the probability of line overloads kept small.

$$\min_{\bar{p},\alpha} \mathbb{E}[c(\bar{p})]$$

$$\text{s.t.} \sum_{i \in G} \alpha_i = 1,\ \alpha \geq 0$$

$$B\delta = \alpha, \delta_n = 0$$

$$\sum_{i \in G} \bar{p}_i + \sum_{i \in W} \mu_{\mathbf{i}} = \sum_{i \in D} d_i$$

$$\bar{f}_{ij} = \beta_{ij}(\bar{\theta}_i - \bar{\theta}_j),$$

$$B\bar{\theta} = \bar{p} + \mu - d,\ \bar{\theta}_n = 0$$

$$s_{ij}^2 \geq \beta_{ij}^2 \sum_{k \in W} \sigma_{\mathbf{k}}^{\mathbf{2}} (B_{ik}^+ - B_{jk}^+ - \delta_i + \delta_j)^2$$

$$|\bar{f}_{ij}| + s_{ij}\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$$

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$$|\overline{f}_{ij}| + s_{ij}\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$$

A convex optimization problem.

Big cases

Polish 2003-2004 winter peak case

- 2746 buses, 3514 branches, 8 wind sources
- 5% penetration and $\sigma = .3\mu$ each source

CPLEX: the optimization problem has

- 36625 variables
- 38507 constraints, 6242 conic constraints
- 128538 nonzeros, 87 dense columns

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Big cases

CPLEX:

- total time on 16 threads = 3393 seconds
- "optimization status 6"
- solution is wildly infeasible

Gurobi:

- time: 31.1 seconds
- "Numerical trouble encountered"

Cutting-plane method

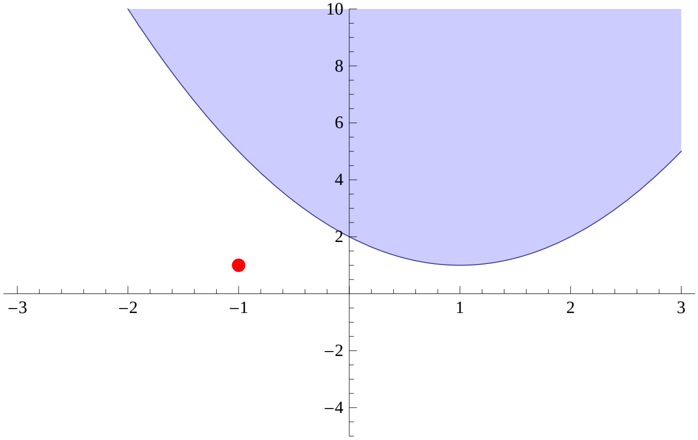
overview

Cutting-plane algorithm:

```
remove all conic constraints
repeat until convergence:
    solve linearly constrained problem
    if no conic constraints violated: return
    find separating hyperplane for maximum violation
    add linear constraint to problem
```

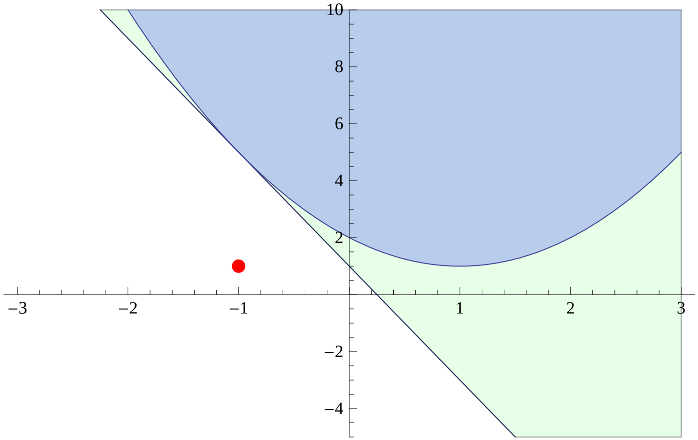
Cutting-plane method

Candidate solution violates conic constraint



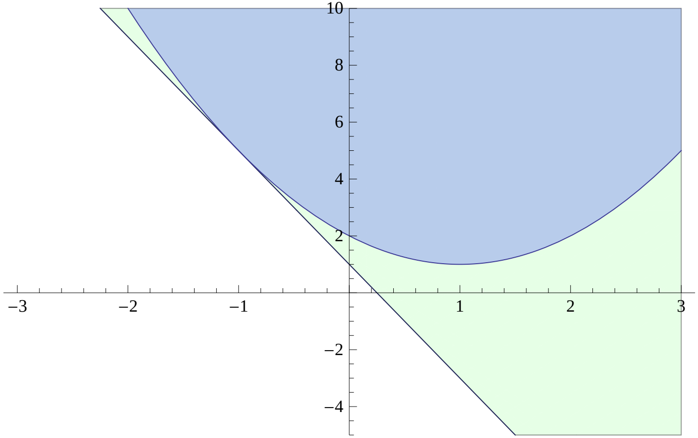
Cutting-plane method

Separate: find a linear constraint also violated



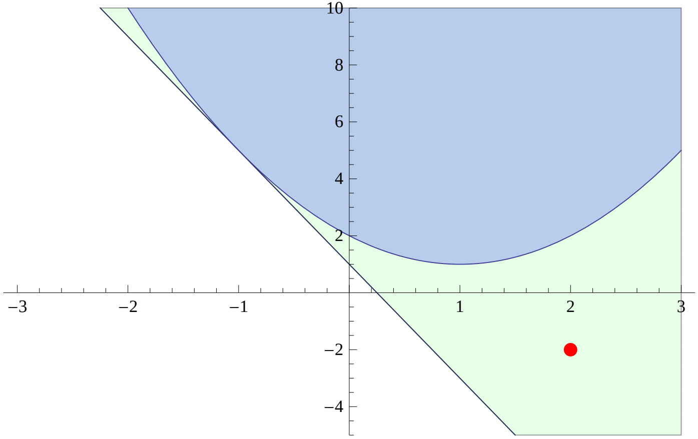
Cutting-plane method

Solve again with linear constraint



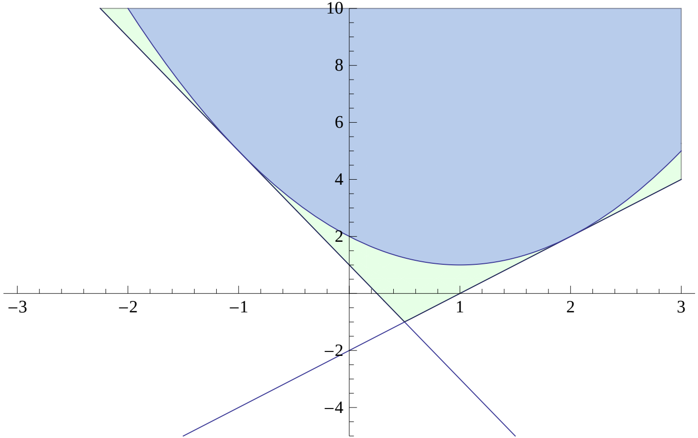
Cutting-plane method

New solution still violates conic constraint



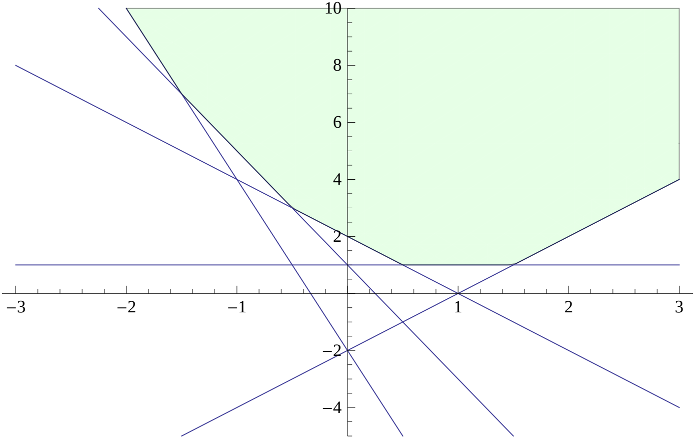
Cutting-plane method

Separate again



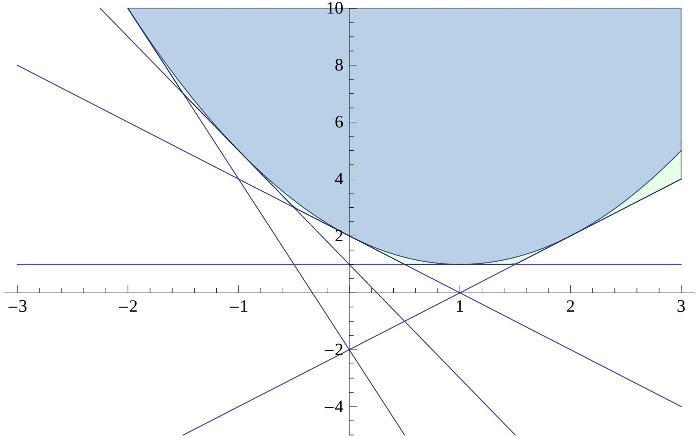
Cutting-plane method

We might end up with many linear constraints

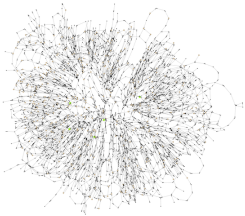


Cutting-plane method

... which approximate the conic constraint



Polish 2003-2004 case
CPLEX: “opt status 6”
Gurobi: “numerical trouble”



Example run of cutting-plane algorithm:

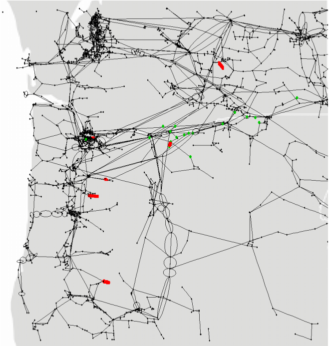
Iteration	Max rel. error	Objective
1	1.2e-1	7.0933e6
4	1.3e-3	7.0934e6
7	1.9e-3	7.0934e6
10	1.0e-4	7.0964e6
12	8.9e-7	7.0965e6

Total running time: 32.9 seconds

Back to motivating example

BPA case

- standard OPF: cost 235603, 7 lines unsafe $\geq$ 8% of the time
- CC-OPF: cost 237297, every line safe $\geq$ 98% of the time
- run time = 9.5 seconds (one cutting plane!)



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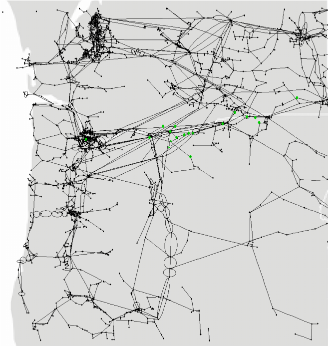

Summary: Bienstock, Chertkov, Harnett 2012

- Specialized cutting-plane algorithm proves effective
- Commercial solvers do not
- Algorithm efficient even in cases with thousands of buses/lines
- Algorithm can be made robust with respect to data errors

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Can we handle power flows more accurately?

Active power, lossless OPF:

$$\min_{p,\theta} \; c(p)$$

s.t.

$$\sum_{j:ij \in \mathcal{L}} \beta_{ij} \sin(\theta_i - \theta_j) \; = \; p_i \; - \; d_i \qquad \forall i \in \mathcal{B} \tag{4}$$

$$|\beta_{ij} \sin(\theta_i - \theta_j)| \; \leq \; u_{ij} \quad \text{for each line } ij \tag{5}$$

$$P_g^{min} \; \leq \; p_g \; \leq \; P_g^{max} \quad \text{for each generator } g \tag{6}$$

From Boyd (2012): Suppose you solve the **convex optimization problem**:

$$\min \sum_{ij \in \mathcal{L}} \beta_{ij} \Psi(\rho_{ij})$$

s.t.

$$\sum_{j: ij \in \mathcal{L}} \beta_{ij} \rho_{ij} \; - \; \sum_{j: ji \in \mathcal{L}} \beta_{ij} \rho_{ji} \; = \; p_i \; - \; d_i \qquad \forall i \in \mathcal{B} \qquad (7)$$

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How can we incorporate this methodology into OPF-type problems?

Suppose you solve the **convex optimization problem**:

$$\min_{p,\rho,\delta \geq 0} \; c(p) \; + \; D \sum_{ij \in \mathcal{L}} \beta_{ij} \Psi(\rho_{ij}) \; - \; K \sum_{ij \in \mathcal{L}} \beta_{ij} \log(\delta_{ij}) \tag{9}$$

s.t.

$$\sum_{j:ij \in \mathcal{L}} \beta_{ij} \rho_{ij} \; - \; \sum_{j:ji \in \mathcal{L}} \beta_{ij} \rho_{ji} \; = \; p_i \; - \; d_i \qquad \forall i \in \mathcal{B} \tag{10}$$

$$|\rho_{ij}| \; + \; \min\{1, u_{ij}/\beta_{ij}\}\delta_{ij} < \; \min\{1, u_{ij}/\beta_{ij}\} \quad \text{for each line } ij \tag{11}$$

$$P_g^{min} \; \leq \; p_g \; \leq \; P_g^{max} \quad \text{for each generator } g$$

For **appropriate** positive constants **D** (small) and **K** (large).

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(1) The optimal ρ_{ij} are approximate optimal active flows

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(2) $\rho_{ij} \approx \sin(\theta_i - \theta_j)$ $\quad \theta =$ optimal duals to (10).

Somewhat more general: $\gamma_{\mathbf{ij}}$ = sine of max phase difference on **ij**

$$\min_{p,\rho,\delta \geq 0} \; c(p) \; + \; D \sum_{ij \in \mathcal{L}} \beta_{ij} \Psi(\rho_{ij}) \; - \; K \sum_{ij \in \mathcal{L}} \beta_{ij} \log(\delta_{ij}) \tag{12}$$

s.t.

$$\sum_{j:ij \in \mathcal{L}} \beta_{ij} \rho_{ij} \; - \; \sum_{j:ji \in \mathcal{L}} \beta_{ij} \rho_{ji} \; = \; p_i \; - \; d_i \qquad \forall i \in \mathcal{B} \tag{13}$$

$$|\rho_{ij}| \; + \; \min\{\gamma_{ij}, u_{ij}/\beta_{ij}\}\delta_{ij} < \; \min\{\gamma_{ij}, u_{ij}/\beta_{ij}\} \quad \text{for each line } ij \tag{14}$$

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Ongoing work:

$$\min_{p,\rho,\delta\geq 0} \; c(p) \; + \; D\sum_{ij\in\mathcal{L}} \beta_{ij}\Psi(\rho_{ij}) \; - \; K\sum_{ij\in\mathcal{L}} \beta_{ij}\log(\delta_{ij})$$

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- Outer envelope approximation to functions $\mathbf{c}, \;\; \mathbf{\Psi}, \;\; -\mathbf{log}$
- $\mathbf{D} \rightarrow \mathbf{0}, \;\; \mathbf{K} \rightarrow +\infty$ needs to be managed
- Existing methodology for logarithmic barrier algorithms can be leveraged
- Early infeasibility detection can be important

Dörfler, Chertkov, Bullo 2013: an approximation

$$\min_{p,\vartheta} \; c(p)$$

s.t.

$$\sum_{j:ij \in \mathcal{L}} \beta_{ij}(\vartheta_i - \vartheta_j) \;=\; p_i \;-\; d_i \qquad \forall i \in \mathcal{B}$$

$$|\vartheta_i - \vartheta_j| < \; \min\{1, u_{ij}/\beta_{ij}\} \quad \text{for each line } ij$$

- The ϑ are auxiliary variables only
- In experiments, $\vartheta_i - \vartheta_j$ provides a close approximation to the lossless (active) AC power flow on each line ij
- (But does not provide phase angles)

Incorporation into chance-constrained problem:

A combination of two ideas

- On any line ij, we replace $\sin(\theta_i - \theta_j)$ with the quantity $\vartheta_i - \vartheta_j$

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- But in **either** case the constraint is stochastic

Chance-constrained version: $P(|\vartheta_i - \vartheta_j| > \gamma_{ij}) < \epsilon_{ij}$

Example: $\epsilon_{ij} = 10^{-4}$.

Control (again)

For each generator i, two parameters:

- $\overline{p_i}$ = mean output
- α_i = response parameter

Real-time output of generator i:

$$p_i = \overline{p}_i - \alpha_i \sum_j \Delta\omega_j$$

where $\Delta\omega_j$ = change in output of renewable j (from mean).

$$\sum_i \alpha_i = 1$$

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So for any line ij, $\boldsymbol{\vartheta}_\mathbf{i} - \boldsymbol{\vartheta}_\mathbf{j} = \sum_k a_k(\bar{p}_k - d_k + \mu_k) \ + \ \sum_k b_k \boldsymbol{\omega}_\mathbf{k}$

Chance-constrained, thermal and sync-aware (approximate) OPF: Choose mean generator outputs and control to minimize expected cost, with the probability of line overloads and phase angle excursions kept small. (abridged)

$$\min_{\overline{p},\alpha} \mathbb{E}[c(\overline{p})]$$

$$\text{s.t.} \sum_{i \in G} \alpha_i = 1, \;\; \alpha \geq 0$$

$$B\delta = \alpha$$

$$\sum_{ij \in \mathcal{L}} \beta_{ij}(\bar{\vartheta}_i - \bar{\vartheta}_j) = \overline{p_i} + \mu_{\mathbf{i}} - d_i$$

$$P(\beta_{ij}|\boldsymbol{\vartheta}_{\mathbf{i}} - \boldsymbol{\vartheta}_{\mathbf{j}}| > u_{ij}) \;\; \leq \epsilon_1 \quad \text{for each line } ij$$

$$P(|\boldsymbol{\vartheta}_{\mathbf{i}} - \boldsymbol{\vartheta}_{\mathbf{j}}| > \gamma_{ij}) \;\; \leq \epsilon_2 \quad \text{for each line } ij$$

$$P(\mathbf{p_g} < P_g^{min} \text{ or } P_g^{max} < \mathbf{p_g}) \leq \epsilon_3 \quad \text{for each generator } g$$

$\boldsymbol{\epsilon_2 \ll \epsilon_3 \ll \epsilon_1}$

Again: a conic optimization problem

Summary of computational experiments

- On Polish grid example (approximately 3000 buses, 388 generators and 3799 lines), cutting-plane algorithm converges within 5-30 seconds and 2-30 iterations on a current computer
- Algorithm 'discovers' at-risk lines
- Fairly smooth convergence with decreased risk as the generation dispatch and control parameters are improved
- Geographical patterns of at-risk lines exposed
- Standard OPF produces poor solutions – risky and expensive
- See paper!